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Satellite Application Facility
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AAPP DOCUMENTATION

ANNEX OF SCIENTIFIC DESCRIPTION

AAPP NAVIGATION

Anne Marsouin (Météo-France)

Pascal Brunel (Météo-France)

This documentation was developed within the context of the EUMETSAT Satellite Application Facility on Numerical Weather Prediction (NWP SAF), under the Cooperation Agreement dated 1st December 2003, between EUMETSAT and the Met Office, UK, by one or more partners within the NWP SAF. The partners in the NWP SAF are the Met Office, ECMWF, KNMI and Météo France.

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Version	Date	Author / changed by	Remarks
1.0	June 2004	A. Marsouin, P. Brunel	Original
1.1	07 April 2005	N. Atkinson	Add section 8
1.2	June 2006	N. Atkinson	Change title and introduction text for AAPP v6
1.3	Oct 2011	N. Atkinson	Update logos, for AAPP v7

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1. INTRODUCTION

Versions 1 to 5 of the ATOVS and AVHRR Processing Package (AAPP) support the processing of direct readout HRPT data from the NOAA polar orbiter satellites. To prepare for the launch of METOP (in 2006), the AAPP navigation modules were revised in AAPP v5. This document details the navigation method that is used in AAPP v5 and subsequent versions of AAPP.

The basic algorithm is the same as in the earlier AAPP versions and has been presented in:

[1] Brunel P. and Marsouin A., 2000, Operational AVHRR navigation results, *International Journal of Remote Sensing*, Vol. 21, No. 5, 951-972.

[2] Rosborough G.W., Baldwin D. and Emery W., 1994, Precise AVHRR Image Navigation, *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 32, No. 3, May 1994, 644-657.

The new version differs on the following points:

- several modes of the satellite attitude can be used,
- several scanning geometry are considered (plane, conic, IASI)
- the software is more modular and a new satellite or instrument can be added easily

This text gives a detailed description of the equations used in AAPP version 5 and subsequent versions, and indicates the names of the corresponding AAPP subroutines.

AAPP contains subroutines that are specific to AVHRR navigation:

- invloc and invlorb, to convert the latitude and longitude of the viewed pixel into time and scanning angle,
- estimatt and estimrp, to estimate the attitude error using landmarks true positions and observed viewing vectors.

These routines have not been revisited but only modified concerning the calls to other subroutines. They are not considered here. The navigation subroutines that have not been changed are also ignored in this text.

2. DEFINITIONS, UNITS, CONVENTIONS

The equations presented in this document assume the following units: angle in radians, time in seconds and distance in kilometers.

A vector is written $\boldsymbol{V}$, $\boldsymbol{SM}$ or, if it is a unit vector, $\boldsymbol{v}$, $\boldsymbol{sm}$. A matrix is written **M**. The components of a vector $\boldsymbol{v}$ on the X, Y, Z axis of a reference frame are *v(1)*, *v(2)*, *v(3)*, in order to be close to the notations of the FORTRAN code.

In the formula $\boldsymbol{w} = \boldsymbol{u} \text{ x } \boldsymbol{v}$, x indicates a vector product, which corresponds to :

$$w(1) = u(2) \cdot v(3) - u(3) \cdot v(2)$$

$$w(2) = u(3) \cdot v(1) - u(1) \cdot v(3)$$

$$w(3) = u(1) \cdot v(2) - u(2) \cdot v(1)$$

Defining a cone

A cone is used for several applications: conical scanning, footprint calculation and IASI viewing geometry. So, a few definitions relative to a cone are given here.

- A cone is defined by an axis and a half angle, δ.
- Using a reference frame where the center is at the cone apex, the X-axis is aligned with the cone axis and Y- and Z axis are normal to the cone axis, any vector $\boldsymbol{V}$ lying in the cone can be written:

$$\boldsymbol{V} = \begin{bmatrix} \cos(\delta) \\ \sin(\delta) \cdot \cos(\phi) \\ \sin(\delta) \cdot \sin(\phi) \end{bmatrix}$$

  where ϕ is the angle between the plane (X,Y) and the plane (X,$\boldsymbol{V}$), varying from 0 to 2π. This angle has no specific name (unlike the half angle). As it is similar to an azimuth, this name will be used in the document.

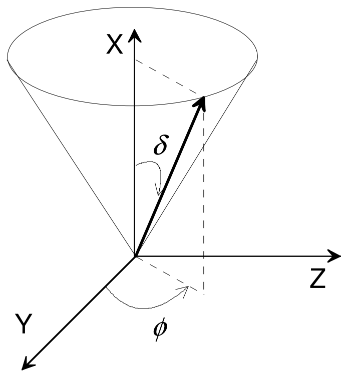

Figure 2-1: vector lying in a cone

Reference frames and conversion matrix

The reference frames used for navigation have a physical meaning and each axis direction can be precisely defined. However, its orientation can be chosen arbitrarily, as long as the reference frame is direct. The orientations already used in AAPP have been kept; they correspond to those of the NOAA documents.

The conversion matrix $\mathbf{M}_{\mathbf{old\_to\_new}}$ from a reference frame R_{old} to a reference frame R_{new} is the matrix that converts the coordinates in R_{old} of a vector $\boldsymbol{V}$ into its coordinates in R_{new}

$$\boldsymbol{V}_{\mathbf{new}} = \mathbf{M}_{\mathbf{old\_to\_new}} \times \boldsymbol{V}_{\mathbf{old}}$$

The R_{new} unit vectors in R_{old} are the columns of $\mathbf{M}_{\mathbf{new\_to\_old}}$

the lines of $\mathbf{M}_{\mathbf{old\_to\_new}}$

For all conversion matrices used here: $\mathbf{M^{-1}=M^{T}}$

If R_{new} is obtained from R_{old} by a rotation of angle φ about one axis of R_{old}, the conversion matrices between R_{new} and R_{old} are given by two of the six formulas:

rotation about X axis rotation about Y axis rotation about Z-axis

$$\mathbf{M}_{\mathbf{new\_to\_old}}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & -\sin(\varphi) \\ 0 & \sin(\varphi) & \cos(\varphi) \end{bmatrix} \quad \begin{bmatrix} \cos(\varphi) & 0 & \sin(\varphi) \\ 0 & 1 & 0 \\ -\sin(\varphi) & 0 & \cos(\varphi) \end{bmatrix} \quad \begin{bmatrix} \cos(\varphi) & -\sin(\varphi) & 0 \\ \sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M}_{\mathbf{old\_to\_new}}: \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi) & \sin(\varphi) \\ 0 & -\sin(\varphi) & \cos(\varphi) \end{bmatrix} \quad \begin{bmatrix} \cos(\varphi) & 0 & -\sin(\varphi) \\ 0 & 1 & 0 \\ \sin(\varphi) & 0 & \cos(\varphi) \end{bmatrix} \quad \begin{bmatrix} \cos(\varphi) & \sin(\varphi) & 0 \\ -\sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. REFERENCE FRAMES AND CONVERSION MATRIX

3.1. GREENWICH REFERENCE FRAME (R_G)

R_G is an Earth fixed reference frame.

- The center is at the Earth's gravity center, O.
- X_G is the line from the Earth's gravity center to the intersection between the equator and the Greenwich meridian, which defines the positive direction.
- Z_G is the polar axis, positive toward the North Pole.
- Y_G is normal to X_G and Z_G (in the equatorial plane, toward longitude 90 E).

The following relations are useful:

$$\boldsymbol{OM} = R \cdot \begin{bmatrix} \cos(la_C)\cdot\cos(lo) \\ \cos(la_C)\cdot\sin(lo) \\ \sin(la_C) \end{bmatrix} \qquad \tan(la_C) = \tan(la_G)\cdot\frac{R_P^2}{R_E^2}$$

$$R = \frac{R_P}{\sqrt{1-e^2\cdot\cos^2(la_C)}} \qquad e^2 = 1-\frac{R_P^2}{R_E^2}$$

with

M	: a point on the Earth's surface
lo	: longitude of M, i.e. angle from the plane (polar axis, Greenwich meridian) to the plane (polar axis, meridian of M)
la_G	: geographical latitude of M, i.e. angle from the equatorial plane to the normal to the Earth ellipsoid
la_C	: geocentric latitude of M, i.e. angle from the equatorial plane to OM
R	: Earth's radius at M
R_E , R_P	: Earth's equatorial and polar radius; AAPP uses the reference ellipsoid

GRS 80: R_E = 6378.137 km flattening factor f=1 / 298.25722

$\Rightarrow$ R_P= (1 - f) R_E = 6356.752 km

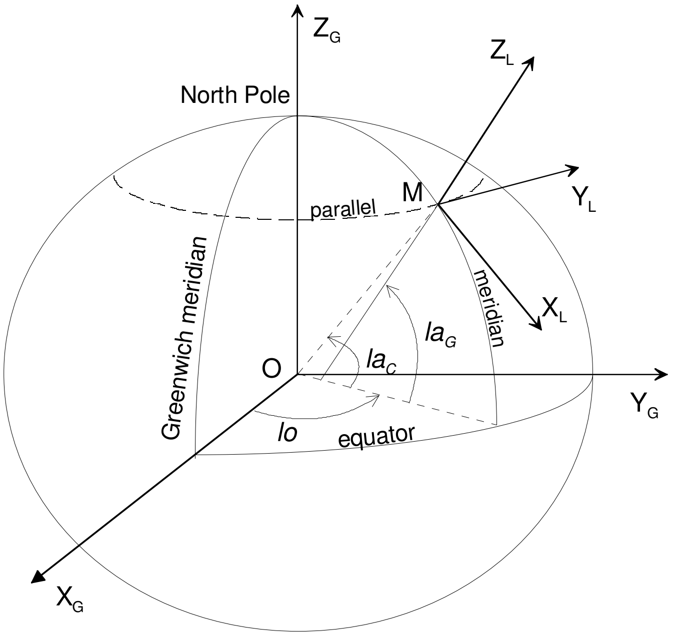

Figure 3-1: R_G and R_L reference frames

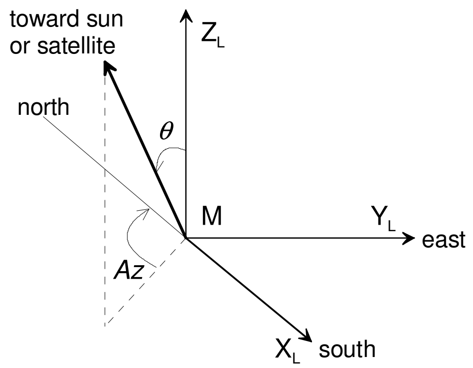

Figure 3-2: zenith and azimuth angles

3.2. LOCAL REFERENCE FRAME OF AN OBSERVATION POINT (R_L)

R_L is an Earth fixed reference frame.

- The center is at the observation point M.
- Z_L is the local vertical (i.e. the normal to the Earth ellipsoid), positive upward
- X_L is normal to Z_L and in the half plane that contains the meridian of the point, positive toward south.
- Y_L is normal to X_L and Z_L.

R_L is deduced from R_G by two rotations

- rotation of an angle *lo* about the axis Z_G, which gives an intermediate reference frame
- rotation of an angle $\pi/2 - la_G$ about the Y-axis of the intermediate reference frame

So, the conversion matrix **L** from R_G to R_L is:

$$\mathbf{L} = \begin{bmatrix} \cos(\pi/2 - la_G) & 0 & -\sin(\pi/2 - la_G) \\ 0 & 1 & 0 \\ \sin(\pi/2 - la_G) & 0 & \cos(\pi/2 - la_G) \end{bmatrix} \mathrm{x} \begin{bmatrix} \cos(lo) & \sin(lo) & 0 \\ -\sin(lo) & \cos(lo) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} \sin(la_G) & 0 & -\cos(la_G) \\ 0 & 1 & 0 \\ \cos(la_G) & 0 & \sin(la_G) \end{bmatrix} \mathrm{x} \begin{bmatrix} \cos(lo) & \sin(lo) & 0 \\ -\sin(lo) & \cos(lo) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} \sin(la_G)\cdot\cos(lo) & \sin(la_G)\cdot\sin(lo) & -\cos(la_G) \\ -\sin(lo) & \cos(lo) & 0 \\ \cos(la_G)\cdot\cos(lo) & \cos(la_G)\cdot\sin(lo) & \sin(la_G) \end{bmatrix}$$

R_L is used only to calculate the viewing angle of an object, sun or satellite, S, seen from an observation point on Earth, M:

- the zenith angle, θ, which is the angle between the local vertical and the point-to-object vector, varies from 0 to 180 degrees.
- the azimuth angle, *Az*, which is the angle between a vertical half plane containing the north direction (i.e. opposite to X_L) and the point-to-object vector (see Figure 3-2); this angle varies from –180 to 180 degrees, is 0 at true north and positive toward east.
  The azimuth angle definition is the one of the NOAA level 1b.
  Due to R_L axis orientation, *Az* is the angle ***from*** the point-to-object vector ***to*** the vertical half plane containing the north direction.

The vector ***MS*** can be expressed in R_L:

$$\boldsymbol{MS} = D \begin{bmatrix} \sin(\theta)\cdot\cos(\pi - Az) \\ \sin(\theta)\cdot\sin(\pi - Az) \\ \cos(\theta) \end{bmatrix} = D \begin{bmatrix} \sin(\theta)\cdot\cos(Az) \\ -\sin(\theta)\cdot\sin(Az) \\ \cos(\theta) \end{bmatrix}$$

where *D* is the distance from the observation point to the object.

The conversion matrix **L** is calculated in two subroutines: **zenazi**, the general routine to calculate the viewing angles and **initrack**, which is adapted to the case of a tracking station.

3.3. NOMINAL ATTITUDE REFERENCE FRAME (R_A)

R_A represents to the nominal orientation of the spacecraft, i.e. the attitude that the spacecraft would have if the attitude control system was perfect. The precise definition of this frame depends on the current attitude mode, “local normal pointing” for NOAA POES and “yaw steering” for METOP. However, the various attitude modes of the polar orbiter satellites are not so different and a general definition can be given for R_A :

- The center is at the satellite gravity center.
- X_A is close to the local terrestrial vertical, positive toward the Earth.
- Y_A is nearly collinear to the satellite velocity, opposite in direction.
- Z_A is nearly orthogonal to the orbit plane, positive in the direction of the angular momentum vector of the orbit i.e toward the left of the trajectory.

The following sections precisely define R_A and calculate the conversion matrix **T** from R_A to R_G , for several attitude modes. The matrix **T** is obtained by calculating ***u***, ***v***, ***w***, the unit vectors of R_A in R_G.

$$\mathbf{T} = \begin{bmatrix} u(1) & v(1) & w(1) \\ u(2) & v(2) & w(2) \\ u(3) & v(3) & w(3) \end{bmatrix}$$

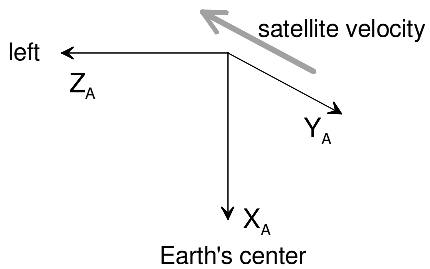

Figure 3-3: R_A reference frame

3.3.1. Local normal pointing mode (R_{A0})

R_{A0} is defined as follows:

- X_{A0} is aligned with the local terrestrial vertical (i.e. the normal to the Earth ellipsoid passing by the satellite), positive toward the Earth.
- Z_{A0} is normal to X_{A0} and to the satellite velocity; it is positive in the direction of the angular momentum vector of the orbit.
- Y_{A0} completes the orthogonal system

The vector ***u*** is calculated with the satellite longitude, *lon*, and geographical latitude lat_G

$$\boldsymbol{u} = \begin{bmatrix} -\cos(lat_G)\cdot\cos(lon) \\ -\cos(lat_G)\cdot\sin(lon) \\ -\sin(lat_G) \end{bmatrix}$$

The vector ***w*** is normal to ***u*** and to the satellite velocity vector ***V***. This vector is the “absolute” velocity or the velocity in an inertial reference frame (as usual orbit prediction).

$$\boldsymbol{w} = -\boldsymbol{u} \times \boldsymbol{V} / \| \boldsymbol{u} \times \boldsymbol{V} \|$$

And finally: $\boldsymbol{v} = \boldsymbol{w} \times \boldsymbol{u}$

3.3.2. Yaw steering mode (R_{A1})

R_{A0} and R_{A1} have the same X-axis and they differ on the Y- and Z-axis:

- X_{A1} is aligned with the local terrestrial vertical (i.e. the normal to the Earth ellipsoid passing by the satellite), positive toward the Earth.
- Z_{A1} is normal to X_{A1} and to the satellite velocity relative to the Earth, i.e. the velocity in R_G; it is positive in the direction of the angular momentum vector of the orbit.
- Y_{A1} completes the orthogonal system

The calculations are the same as in 3.3.1 except for the vector ***w*** is normal to ***u*** and to the satellite velocity vector, ***Vr*** , in the reference frame R_G.

$$\boldsymbol{w} = - \boldsymbol{u} \times \boldsymbol{Vr} \,/\, \| \boldsymbol{u} \times \boldsymbol{Vr} \|$$

Vr is a relative velocity vector, obtained as follows:

$$\boldsymbol{Vr} = \boldsymbol{V} - \boldsymbol{\varpi}_E \times \boldsymbol{OS} \quad \Leftrightarrow \quad \begin{aligned} Vr_{R_G}(1) &= V_{R_G}(1) + \omega_E \cdot OS(2)_{R_G} \\ Vr_{R_G}(2) &= V_{R_G}(2) - \omega_E \cdot OS(1)_{R_G} \\ Vr_{R_G}(3) &= V_{R_G}(3) \end{aligned}$$

with $\boldsymbol{\varpi}_E$: angular velocity vector of the Earth

$\boldsymbol{\varpi}_E = 7.292115147\ 10^{-5}$ rad/s (i.e. 6.300387487 rad/day)

OS : vector from Earth's gravity centre to satellite

3.3.3. Geocentric mode (R_{A2})

R_{A2} is defined as follows:

- X_{A2} is aligned with the line from the Earth center to the satellite center, i.e. the geocentric vertical, positive toward the Earth.
- Z_{A2} is normal to X_{A2} and to the satellite velocity, i.e. orthogonal to the orbit plane; it is positive in the direction of the angular momentum vector of the orbit.
- Y_{A2} completes the orthogonal system

The calculations are simpler: ***u*** directly depends on the satellite Cartesian coordinates and ***w*** is normal to ***u*** and to the satellite velocity vector ***V***.

$$\boldsymbol{u} = - \boldsymbol{OS} \,/\, \| \boldsymbol{OS} \|$$

$$\boldsymbol{w} = - \boldsymbol{u} \times \boldsymbol{V} \,/\, \| \boldsymbol{u} \times \boldsymbol{V} \|$$

$$\boldsymbol{v} = \boldsymbol{w} \times \boldsymbol{u}$$

3.3.4. *V*elocity calculations

It should be noted that the velocity vectors stored in the "satpos" files are relative velocity vectors in R_G . They have been calculated by one of the subroutines **pvitogrw**, **pvi50togrw** or **pvitegrw**.

The matrix **T** is calculated by the subroutine **snagre**, which use a "satpos" velocity vector as input. So, **snagre** does not convert ***V*** into ***Vr*** but ***Vr*** into ***V***, when needed (NOAA operational mode and geocentric mode):

$$\Leftrightarrow \quad V_{R_G}(1) = Vr_{R_G}(1) - \omega_E \cdot OS(2)_{R_G}$$
$$V_{R_G}(2) = Vr_{R_G}(2) + \omega_E \cdot OS(1)_{R_G}$$
$$V_{R_G}(3) = Vr_{R_G}(3)$$

3.4. SPACECRAFT-FIXED REFERENCE FRAME (R_S)

R_S is defined as follows:

- The center is at the satellite gravity center.
- X_S is the "vertical" axis of the spacecraft.
- Y_S is the "longitudinal" axis of the spacecraft.
- Z_S is the "transversal" axis of the spacecraft.

These axes are defined precisely with respect to some elements of the spacecraft structure, their positive direction being consistent with the spacecraft nominal orientation.

R_S should be perfectly aligned with R_A, but it differs from it by three small rotation angles: yaw about X_A, roll about Y_A and pitch about the Z_A. Using the small angle approximation, the conversion matrix between R_S and R_A are given by simple formulas.

conversion from R_S to R_A

$$\mathbf{A} = \begin{bmatrix} 1 & p & -r \\ -p & 1 & y \\ r & -y & 1 \end{bmatrix}$$

conversion from R_A to R_S

$$\mathbf{A^T} = \begin{bmatrix} 1 & -p & r \\ p & 1 & -y \\ -r & y & 1 \end{bmatrix}$$

where *y, r,* and *p* are the yaw, roll and pitch angles.

The sign conventions are the following:

yaw > 0 : the spacecraft longitudinal axis is orientated toward the right of the trajectory
roll > 0 : the spacecraft vertical axis is on the right of the sub-track
pitch > 0 : the spacecraft vertical axis is behind the sub-point

The attitude matrix **A** is calculated in the subroutine **calatt**

3.5. INSTRUMENT REFERENCE FRAME (R_I)

Each instrument is mounted on the spacecraft but some misalignments may occur. So, the instrument-fixed reference frame R_I, which should be perfectly aligned with R_S, may differ slightly from R_S it by three small misalignment angles.

R_I is defined as follows:

- The center is at the satellite gravity center.
- X_I is nearly aligned with X_s.
- Y_I is nearly aligned with Y_s.
- Z_I is nearly aligned with Z_s.

R_I is obtained from R_s by 3 rotations: in yaw about X_S, in roll about Y_S and in pitch about Z_S.

The conversion matrices are the following:

conversion from R_I to R_S

$$\mathbf{D} = \begin{bmatrix} 1 & p_I & -r_I \\ -p_I & 1 & y_I \\ r_I & -y_I & 1 \end{bmatrix}$$

conversion from R_S to R_I

$$\mathbf{D^T} = \begin{bmatrix} 1 & -p_I & r_I \\ p_I & 1 & -y_I \\ -r_I & y_I & 1 \end{bmatrix}$$

where y_I, r_I and p_I are the misalignment angles in yaw, roll and pitch.

3.6. INTERMEDIATE REFERENCE FRAMES

Additional reference frames are used for some instruments. They are defined below.

R_1, useful for an instrument inclined forward or backward of the satellite nadir, is deduced from R_I by a rotation of an angle β about the instrument transversal axis Z_I:

- The center is at the satellite gravity center.
- X_1 is obtained from X_I by a rotation of β about Z_I.
- Y_1 is obtained from Y_I by a rotation of β about Z_I
- $Z_1 = Z_I$.

The conversion matrix from R_1 to R_I is;

$$\mathbf{M_1} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

β is counted positively according to the direct orientation of R_I, which means:

$\beta > 0$ X_1 looks backwards of the satellite sub-point (same convention as for the roll)

R_2, useful to define IASI scanning and to calculate the instrument footprint, has its X-axis aligned with the viewing direction. R_2 is deduced from R_1 by a rotation of an angle $-\alpha$ about axis Y_1 (the negative sign is due to the scanning angle definition, see 4.2 for details):

- The center is at the satellite gravity center.
- X_2 is obtained from Z_1 by a rotation of $-\alpha$ about Y_1.
- $Y_2 = Y_1$.
- Z_2 is obtained from Z_1 by a rotation of $-\alpha$ about Y_1

The conversion matrix from R_2 to R_1 is:

$$\mathbf{M_2} = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}$$

α is counted positively according to the direct orientation of R_1, which means:

$\alpha > 0$ X_2 looks on the left of the satellite trajectory

R_3, useful to calculate the instrument footprint in case of conical scanning, has its X-axis aligned with the viewing direction. R_3 is deduced from R_1 by two rotations

- rotation of an angle ϕ about axis X_1, which gives the intermediate reference frame R_{13}
- rotation of an angle δ about axis Z_{13}, which gives the reference frame R_3

The conversion matrix from R_3 to R_1 is:

$$\mathbf{M_3} = \mathbf{M_{13\_to\_1}} \text{ x } \mathbf{M_{3\_to\_13}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \text{x} \begin{bmatrix} \cos(\delta) & -\sin(\delta) & 0 \\ \sin(\delta) & \cos(\delta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{M_3} = \begin{bmatrix} \cos(\delta) & -\sin(\delta) & 0 \\ \sin(\delta)\cdot\cos(\phi) & \cos(\delta)\cdot\cos(\phi) & -\sin(\phi) \\ \sin(\delta)\cdot\sin(\phi) & \cos(\delta)\cdot\sin(\phi) & \cos(\phi) \end{bmatrix}$$

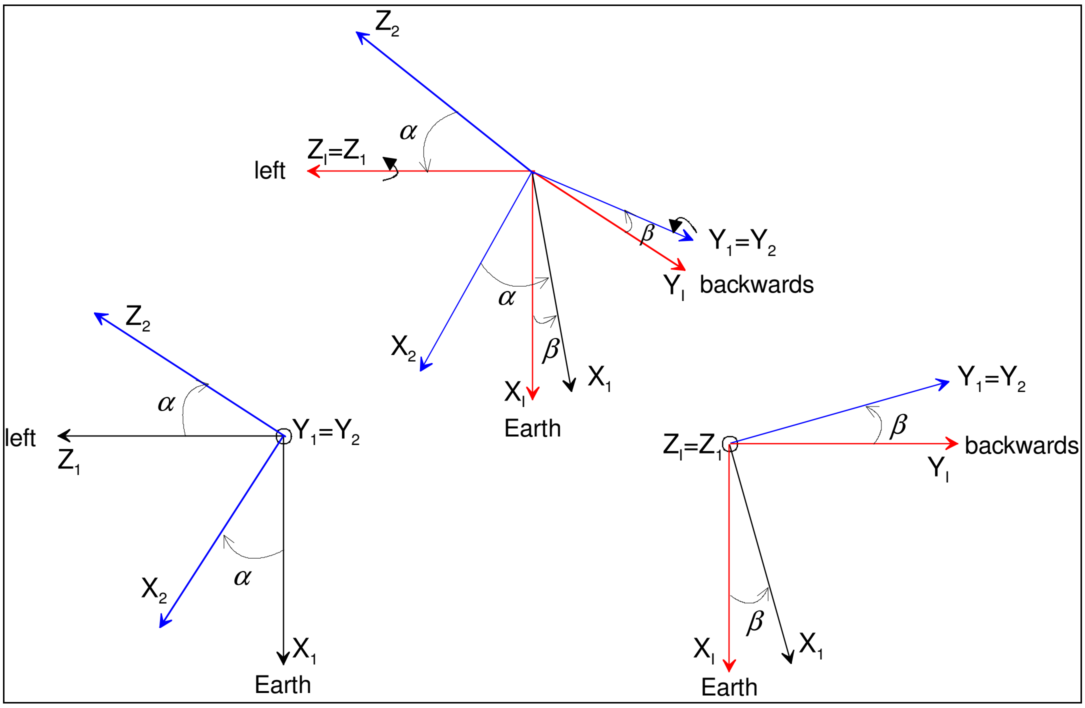

Figure 3-4: R_I, R_1 and R_2 reference frames.

R_I (in red) is rotated by an angle β about the axis Z_I to obtain R_1 (in black), which is rotated by an angle $-\alpha$ about the axis Y_1 to obtain R_2 (in blue).

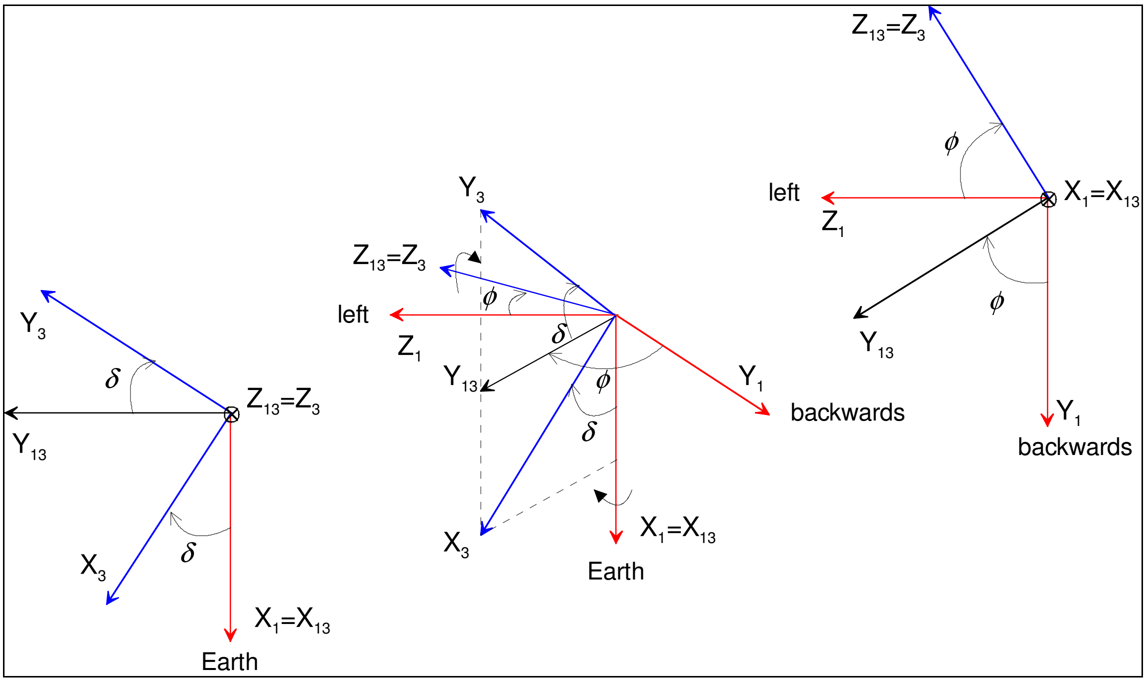

Figure 3-5: R_1 to R_3 conversion.

R_1 (in red) is rotated by an angle ϕ about the axis X_1 to obtain R_{13} (in black), which is rotated by an angle δ about the axis Z_{13} to obtain R_3 (in blue).

In Figure 3-4 and Figure 3-5, the words "left" and "backwards" refer to the satellite trajectory and indicate only approximate directions.

4. SCANNING GEOMETRY

4.1. PRINCIPLES

The viewing direction of the IFOV center (IFOV = Instantaneous Field of View) can be calculated in the spacecraft-fixed reference frame R_S in three stages:

a) the line and pixel numbers are converted into a time and an angle, scanning angle or scanning azimuth,
b) the time and angle gives the unit vector of the direction satellite-toward-viewed-point (***sm***) in the reference frame R_I,
c) the unit vector is converted from R_I to R_S, taken into account the instrument misalignments.

Stages a) and b) depend on the scanning geometry and are described in the following paragraphs. Stage c) is considered as independent of the scanning geometry and made as follows:

$$\boldsymbol{sm}_{\mathbf{R_S}} = \mathbf{D}\, \boldsymbol{sm}_{\mathbf{R_I}} = \begin{bmatrix} 1 & p_I & -r_I \\ -p_I & 1 & y_I \\ r_I & -y_I & 1 \end{bmatrix} \boldsymbol{sm}_{\mathbf{R_I}}$$

$$sm_{R_S}(1) = sm_{R_I}(1) + p_I \cdot sm_{R_I}(2) - r_I \cdot sm_{R_I}(3)$$
$$sm_{R_S}(2) = -p_I \cdot sm_{R_I}(1) + sm_{R_I}(2) + y_I \cdot sm_{R_I}(3)$$
$$sm_{R_S}(3) = r_I \cdot sm_{R_I}(1) - y_I \cdot sm_{R_I}(2) + sm_{R_I}(3)$$

There is not a unique subroutine associated to each subsection of section 4.

lptoviewvect do the calculations presented in 4.2, 4.3 and 4.1
lptoviewvect_iasi do the calculations presented in 4.4 and 4.1

IASI has been considered separately since four pixels are obtained simultaneously, as explained in 4.4.

4.2. SCANNING IN A PLANE

Most radiometers scanning in a plane are cross-track scanners, i.e. they scan perpendicular to the direction of movement of the satellite. The scanning plane may also be tilted backward or forward, by an angle β as defined in 3.6.

The line and pixel numbers are converted into time and scanning angle as follows:

$$t = t_0 + (l - l_0) \cdot T_{stepL} + t_{offset} + (p-1) \cdot T_{stepP}$$

$$\alpha = (p - p_0) \cdot \alpha_{step}$$

with l, l_0 : line number and reference line number

t, t_0 : start times of the line numbers l and l_0

p : pixel number

p_0 : pixel number at sub-track, i.e. (real) pixel number when the viewing direction is aligned with X_1

α : scanning angle, defined as the angle from the viewing direction to the axis Z_1 counted positively according to the direct orientation of R_1

α>0 on the left of the satellite trajectory

α_{step} : center to center FOV step angle (FOV = Field of View),

as the angle α increases from right to left

α_{step} >0 for a radiometer scanning from right to left

T_{stepL} : full scan period, i.e. the time interval between two consecutive lines

T_{stepP} : step time / FOV, i.e. the time interval between two consecutive pixels

t_{offset} : time interval between the start time and the first pixel of the line (initial offset time from TIP to start of integration period, for NOAA)

The above equations and definitions are based on the NOAA satellite scanning instrument parameters. However they are general enough to be applied to other instruments. For NOAA instrument, the time is obtained through the TIP clock (TIP=TIROS Information Processor) onboard the spacecraft.

The viewing direction unit vector, ***sm***, is in the scanning plane (Z_1,X_1) and its angle with the axis Z_1 is $\pi/2-\alpha$:

$$\boldsymbol{sm}_{\mathbf{R_1}} = \begin{bmatrix} \sin(\pi/2-\alpha) \\ 0 \\ \cos(\pi/2-\alpha) \end{bmatrix} = \begin{bmatrix} \cos(\alpha) \\ 0 \\ \sin(\alpha) \end{bmatrix}$$

For a cross-track scanner, β=0, the above formula directly gives viewing vector in R_I. In the general case, the viewing vector is converted from R_1 to R_I with the matrix $\mathbf{M_1}$ defined in 3.6:

$$\boldsymbol{sm}_{\mathbf{R_I}} = \mathbf{M_1}\, \boldsymbol{sm}_{\mathbf{R_1}} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{x} \begin{bmatrix} \cos(\alpha) \\ 0 \\ \sin(\alpha) \end{bmatrix} = \begin{bmatrix} \cos(\beta)\cdot\cos(\alpha) \\ \sin(\beta)\cdot\cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$$

4.3. CONICAL SCANNING

The radiometer viewing direction lies in a cone and rotates about the cone axis at a constant speed. The cone axis is in the instrument longitudinal plane, aligned with the vertical axis or inclined forward or backward, by an angle β (as defined in 3.6). It corresponds to the axis X_1 of R_1. A line corresponds to a limited part of the cone, forward or backward of the cone axis.

The line number is converted into time as in 4.2 and the pixel number is converted into scanning azimuth by the following equations:

$$\gamma = (p - p_0) \cdot \gamma_{step}$$

$\phi = \gamma$ if backward scanning (of the cone axis)

or $\phi = \gamma + \pi$ if forward scanning (of the cone axis)

with
- p : pixel number
- p_0 : pixel number at sub-track, i.e. (real) pixel number when the viewing direction corresponds to X_1 , if backward scanning, or to $-X_1$, if forward scanning
- γ : scanning angle, defined as the angle from the viewing direction at p_0 to the viewing direction, counted positively according to the direct orientation of R_1
- ϕ : scanning azimuth, i.e. angle from the axis Y_1 to the viewing direction counted positively according to the direct orientation of R_1 ($[0, \pi]$ on the left of the satellite trajectory and $[\pi, 2\pi]$ on the right)
- γ_{step} : center to center FOV step angle

With the angle γ and pixel p_0 definitions, γ and γ_{step} have the following signs with respect to the satellite trajectory:

if backward scanning $\gamma>0$ on the left $\gamma_{step}>0$ scanning from right to left

if forward scanning $\gamma>0$ on the right $\gamma_{step}>0$ scanning from left to right

The scanning azimuth and half angle, δ, give the viewing vector in R_1 (as presented in 2):

$$\boldsymbol{sm}_{\mathbf{R_1}} = \begin{bmatrix} \cos(\delta) \\ \sin(\delta)\cdot\cos(\phi) \\ \sin(\delta)\cdot\sin(\phi) \end{bmatrix}$$

If the cone axis is aligned with the instrument vertical, β=0, the above formula directly gives viewing vector in R_I. In the general case, the viewing vector is converted from R_1 to R_I with the matrix $\mathbf{M_1}$ defined in 3.6:

$$\boldsymbol{sm}_{\mathbf{R_I}} = \mathbf{M_1}\,\boldsymbol{sm}_{\mathbf{R_1}} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{x} \begin{bmatrix} \cos(\delta) \\ \sin(\delta)\cdot\cos(\phi) \\ \sin(\delta)\cdot\sin(\phi) \end{bmatrix}$$

$$\boldsymbol{sm}_{\mathbf{R_I}} = \begin{bmatrix} \cos(\beta)\cdot\cos(\delta) - \sin(\beta)\cdot\sin(\delta)\cdot\cos(\phi) \\ \sin(\beta)\cdot\cos(\delta) + \cos(\beta)\cdot\sin(\delta)\cdot\cos(\phi) \\ \sin(\delta)\cdot\sin(\phi) \end{bmatrix}$$

4.4. IASI SCANNING

IASI instrument, Infrared Atmospheric Sounding Interferometer, is a sounder coupled to an integrated imaging subsystem (IIS). IASI field of view contains 4 sounder pixels and an IIS image of 64 by 64 pixels. The operational IASI navigation will be based on a co-registration of IIS and AVHRR, so the AVHRR Earth location will be converted into an IASI Earth location. Such a method has been adopted because IASI is attached on METOP platform via a vibration damping mechanism.

This section is restricted to IASI sounder and simply applies the geometrical laws of the scanning, producing a "draft" navigation.

IASI scanning can be summarized as follows

- a so-called optical axis moves in a plane normal to the spacecraft longitudinal axis,
- a "view" corresponds to 4 pixels, at a given angular distance, ζ , of the optical axis.

A view is considered as instantaneous. Its line and pixel numbers are converted into time of the view and scanning angle of the optical axis, α_{opt}, with the equations of section 4.2. Then the optical axis unit vector, ***so***, is given in R_I by:

$$\boldsymbol{so}_{\mathbf{R_I}} = \begin{bmatrix} \cos(\alpha_{opt}) \\ 0 \\ \sin(\alpha_{opt}) \end{bmatrix}$$

The four pixels lie in a cone. The cone axis is the optical axis, ***so***, and the half angle is the small angle ξ. ***so*** is aligned with X_2 axis. So, the viewing direction of each pixel M_i is obtained in R_2 by:

$$\boldsymbol{sm}_{i_{\mathbf{R_2}}} = \begin{bmatrix} \cos(\zeta) \\ \sin(\zeta)\cdot\cos(\phi_i) \\ \sin(\zeta)\cdot\sin(\phi_i) \end{bmatrix} = \begin{bmatrix} 1 \\ \zeta\cdot\cos(\phi_i) \\ \zeta\cdot\sin(\phi_i) \end{bmatrix}$$

The "IASI scan azimuth" values are dictated by the direct orientation of R_2 and the numbering of the IASI pixels:

$\phi_1 = 7\pi/4$	M_1 on the right and backward of the optical axis
$\phi_2 = 5\pi/4$	M_2 on the right and forward of the optical axis
$\phi_3 = 3\pi/4$	M_3 on the left and forward of the optical axis
$\phi_4 = \pi/4$	M_4 on the left and backward of the optical axis

The scanning plane is normal to the spacecraft longitudinal axis, $R_1 = R_I$ and the viewing vector is converted from R_2 to R_I with the matrix $\mathbf{M_2}$ (defined in 3.6) where the scanning angle concerns the optical axis:

$$\boldsymbol{sm}_{i_{\mathbf{R_I}}} = \mathbf{M_2}\, \boldsymbol{sm}_{i_{\mathbf{R_2}}}$$

$$\boldsymbol{sm}_{i_{\mathbf{R_I}}} = \begin{bmatrix} \cos(\alpha_{opt}) & 0 & -\sin(\alpha_{opt}) \\ 0 & 1 & 0 \\ \sin(\alpha_{opt}) & 0 & \cos(\alpha_{opt}) \end{bmatrix} \begin{bmatrix} 1 \\ \zeta\cdot\cos(\phi_i) \\ \zeta\cdot\sin(\phi_i) \end{bmatrix} = \begin{bmatrix} \cos(\alpha_{opt}) - \zeta\cdot\sin(\phi_i)\cdot\sin(\alpha_{opt}) \\ \zeta\cdot\cos(\phi_i) \\ \sin(\alpha_{opt}) + \zeta\cdot\sin(\phi_i)\cdot\cos(\alpha_{opt}) \end{bmatrix}$$

5. EARTH LOCATION OF THE VIEWED POINT

Earth location of the viewed pixel is the calculation of the IFOV center latitude and longitude or, which is equivalent, of its cartesian coordinates in the reference frame R_G.

The unit vector of viewing direction, ***sm***, is known in the reference frame R_s through the equations presented in section 4. ***sm***, is converted from R_S to R_G by:

$$\boldsymbol{sm}_{\mathbf{R_G}} = \mathbf{T}^{\mathbf{T}}\, \mathbf{A}^{\mathbf{T}}\, \boldsymbol{sm}_{\mathbf{R_S}}$$

The vector ***OS***, Earth's gravity centre to satellite, is known in R_G. The vector ***OM***, Earth's gravity centre to viewed point, is given by:

$$\boldsymbol{OM}_{\mathbf{R_G}} = \boldsymbol{OS}_{\mathbf{R_G}} + D\, \boldsymbol{sm}_{\mathbf{R_G}}$$

where *D*, distance between the satellite and the viewed point, is the only unknown.
The viewed point is in the Earth's ellipsoid, which gives a second order equation in *D*:

$$\frac{OM(1)^2_{R_G}}{R_E^2} + \frac{OM(2)^2_{R_G}}{R_E^2} + \frac{OM(3)^2_{R_G}}{R_P^2} = 1$$

$$\Leftrightarrow \qquad a \cdot D^2 + 2 \cdot b \cdot D + c = 0$$

with $a = sm(1)^2_{R_G} + sm(2)^2_{R_G} + sm(3)^2_{R_G} \cdot \frac{R_E^2}{R_P^2}$

$$b = OS(1)_{R_G} \cdot sm(1)_{R_G} + OS(2)_{R_G} \cdot sm(2)_{R_G} + OS(3)_{R_G} \cdot sm(3)_{R_G} \cdot \frac{R_E^2}{R_P^2}$$

$$c = OS(1)^2_{R_G} + OS(2)^2_{R_G} + OS(3)^2_{R_G} \cdot \frac{R_E^2}{R_P^2} - R_E^2$$

R_E and R_P are respectively the terrestrial equatorial and polar radius

D is the smallest solution of the equation (the other solution corresponds to the point where the viewing direction goes out of the Earth). As the variable *b* is negative, *D* is given by:

$$D = \frac{-b - \sqrt{b^2 - a \cdot c}}{a}$$

With *D*, the vector $\boldsymbol{OM}_{\mathbf{R_G}}$ can be calculated and its cartesian coordinates are converted into latitude and longitude.

The calculations presented in this section are done in the subroutine **earthpix**

6. SATELLITE AND SUN VIEWING ANGLES

The object, satellite or sun, and the point M are known in R_G. The vector ***MS***, point to object, is calculated in R_G then converted to R_L with the conversion matrix **L** (defined in 3.2):

$$\boldsymbol{MS}_{\mathbf{R_G}} = \boldsymbol{OS}_{\mathbf{R_G}} - \boldsymbol{OM}_{\mathbf{R_G}}$$

$$\boldsymbol{MS}_{\mathbf{R_L}} = \mathbf{L}\ \ \boldsymbol{MS}_{\mathbf{R_G}}$$

As indicated in 3.2, $\boldsymbol{MS}_{\mathbf{R_L}}$ is given by:

$$\boldsymbol{MS} = D \begin{bmatrix} \sin(\theta)\cdot\sin(\pi - Az) \\ \sin(\theta)\cdot\cos(\pi - Az) \\ \cos(\theta) \end{bmatrix}$$

where θ is the zenith angle, Az the azimuth angle and D the distance point-to-object.

So, the zenith and azimuth angles are calculated by:

$$\theta = \text{arcos}\left(\frac{MS(3)_{R_L}}{MS}\right)$$

$$Az = \pi \text{ - datan2}\left(MS(2)_{R_L}, MS(1)_{R_L}\right)$$

These calculations are done in the subroutine **zenazi**, for the general case, and by the subroutine **trackang**, for the case of a tracking station.

7. FOOTPRINT CALCULATION

7.1. PRINCIPLES

For a sounder having a rather large IFOV, it may be useful to locate not only the IFOV center but also the footprint, i.e. the surface on Earth corresponding to the IFOV. This section presents an exact calculation of the footprint. Such a method is probably over complicated for most applications but can be used as reference to test approximate formulas.

The IFOV is generally delimited by a cone. The cone axis is the viewing direction of the pixel center and its half angle, ε, is a small angle. ***ε is equal to half of the IFOV width, which is the parameter commonly used to describe the instrument characteristics***.

There is not a unique subroutine associated to each subsection of section 7:

footprint
calculates a footprint in R_I, as presented in 7.1 and 7.2

footprint_iasi
calculates the four IASI footprints in R_I, as presented in 7.3

contour_sondeur_avhrr
calculates a footprint in geographical coordinates then in AVHRR coordinates (call footprint and other routines)

ellipse_sondeur_avhrr
calculates a footprint in AVHRR coordinates, according to the ellipse approximation presented in 7.2 (call footprint and other routines)

contour_iasi_avhrr
calculates the four IASI footprints in AVHRR coordinates (call footprint_iasi and other routines)

7.2. FOOTPRINT FOR A RADIOMETER SCANNING IN A PLANE

The viewing direction of the pixel center, which is the cone axis, is aligned with X_2 axis. Any viewing direction, ***sp***, lying in the cone is given in R_2 by:

$$\boldsymbol{sp}_{\boldsymbol{R_2}} = \begin{bmatrix} 1 \\ \varepsilon \cdot \cos(\psi) \\ \varepsilon \cdot \sin(\psi) \end{bmatrix} \qquad \text{with } \psi \text{ varying from 0 to } 2\pi$$

The viewing vector is converted from R_2 to R_1 with the matrix $\mathbf{M_2}$:

$$\boldsymbol{sp}_{\mathbf{R_1}} = \mathbf{M}_2\, \boldsymbol{sp}_{\mathbf{R_2}} = \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \mathrm{x} \begin{bmatrix} 1 \\ \varepsilon \cdot \cos(\psi) \\ \varepsilon \cdot \sin(\psi) \end{bmatrix}$$

$$\boldsymbol{sp}_{\mathbf{R_1}} = \begin{bmatrix} \cos(\alpha\ ) - \varepsilon \cdot \sin(\psi) \cdot \sin(\alpha) \\ \varepsilon \cdot \cos(\psi) \\ \sin(\alpha) + \varepsilon \cdot \sin(\psi) \cdot \cos(\alpha\ ) \end{bmatrix} \qquad 0 \le \psi \le 2\pi$$

For a cross-track scanner, β=0, the above formula directly gives viewing vector in R_I. In the general case, the viewing vector is converted from R_1 to R_I with the matrix $\mathbf{M_1}$ defined in 3.6:

$$\boldsymbol{sp}_{\mathbf{R_I}} = \mathbf{M_1}\, \boldsymbol{sp}_{\mathbf{R_1}} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathrm{x} \begin{bmatrix} \cos(\alpha\ ) - \varepsilon \cdot \sin(\psi) \cdot \sin(\alpha) \\ \varepsilon \cdot \cos(\psi) \\ \sin(\alpha) + \varepsilon \cdot \sin(\psi) \cdot \cos(\alpha\ ) \end{bmatrix}$$

$$\boldsymbol{sp}_{\mathbf{R_I}} = \begin{bmatrix} \cos(\beta) \cdot [\cos(\alpha\ ) - \varepsilon \cdot \sin(\psi) \cdot \sin(\alpha)] - \sin(\beta)\varepsilon \cdot \cos(\psi) \\ \sin(\beta) \cdot [\cos(\alpha\ ) - \varepsilon \cdot \sin(\psi) \cdot \sin(\alpha)] + \cos(\beta)\varepsilon \cdot \cos(\psi) \\ \sin(\alpha) + \varepsilon \cdot \sin(\psi) \cdot \cos(\alpha\ ) \end{bmatrix} \qquad 0 \le \psi \le 2\pi$$

Earth location of any point on the IFOV border (i.e. the point corresponding to ***sp***) is the same as Earth location of the IFOV center and it follows equations of section 5. If the objective is to obtain the footprint in AVHRR coordinates, the geographical coordinates are then converted into AVHRR line and pixel numbers.

An ellipse is a good approximation of the footprint. The semi-axis in line is the distance between the IFOV center, M, (already calculated in 4.2) and the point Q at ψ=0:

$$\boldsymbol{sq}_{\mathbf{R_I}} = \begin{bmatrix} \cos(\beta) \cdot \cos(\alpha) - \varepsilon \cdot \sin(\beta) \\ \sin(\beta) \cdot \cos(\alpha) + \varepsilon \cdot \cos(\beta) \\ \sin(\alpha) \end{bmatrix} \qquad \boldsymbol{sm}_{\mathbf{R_I}} = \begin{bmatrix} \cos(\beta) \cdot \cos(\alpha) \\ \sin(\beta) \cdot \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$$

The semi-axis in pixel is the distance between the IFOV center and the point Q at $\psi=\pi/2$: However, if the objective is to obtain the footprint in AVHRR coordinates, there is a very simple formula:

$$a_{PIX} = \frac{\varepsilon}{\alpha_{step}(AHRR)} \qquad \text{in number of AVHRR pixels}$$

7.3. FOOTPRINT FOR CONICAL SCANNING

The pixel center viewing direction, which is the cone axis, is aligned with the X_3 axis. of the reference frame R_3 defined in 3.6. Any viewing direction, ***sp***, lying in the cone is given in R_3 by:

$$\boldsymbol{sp}_{R_3} = \begin{bmatrix} 1 \\ \varepsilon \cdot \cos(\psi) \\ \varepsilon \cdot \sin(\psi) \end{bmatrix} \qquad 0 \le \psi \le 2\pi$$

The viewing vector is converted from R_3 to R_1 with the matrix $\mathbf{M_3}$:

$$\boldsymbol{sp}_{\mathbf{R_1}} = \mathbf{M_3}\, \boldsymbol{sp}_{\mathbf{R_3}}$$

$$\boldsymbol{sp}_{\mathbf{R_1}}=\begin{bmatrix}\cos(\delta) & -\sin(\delta) & 0\\ \sin(\delta)\cdot\cos(\phi) & \cos(\delta)\cdot\cos(\phi) & -\sin(\phi)\\ \sin(\delta)\cdot\sin(\phi) & \cos(\delta)\cdot\sin(\phi) & \cos(\phi)\end{bmatrix}\mathrm{x}\begin{bmatrix}1\\ \varepsilon\cdot\cos(\psi)\\ \varepsilon\cdot\sin(\psi)\end{bmatrix}$$

$$\boldsymbol{sp}_{\mathbf{R_1}}=\begin{bmatrix}\cos(\delta)-\varepsilon\cdot\cos(\psi)\cdot\sin(\delta)\\ \sin(\delta)\cdot\cos(\phi)+\varepsilon\cdot[\cos(\psi)\cdot\cos(\phi)\cdot\cos(\delta)-\sin(\psi)\cdot\sin(\phi)]\\ \sin(\delta)\cdot\sin(\phi)+\varepsilon\cdot[\cos(\psi)\cdot\sin(\phi)\cdot\cos(\delta)+\sin(\psi)\cdot\cos(\phi)]\end{bmatrix}\quad 0\le\psi\le 2\pi$$

***If the cone axis is aligned with the instrument vertical, β=0, the above formula directly gives viewing vector in R_I*.** In the general case, the viewing vector is converted from R_1 to R_I with the matrix $\mathbf{M_1}$ defined in 3.6:

$$\boldsymbol{sp}_{\mathbf{R_I}}=\mathbf{M_1}\,\boldsymbol{sp}_{\mathbf{R_1}}=\begin{bmatrix}\cos(\beta) & -\sin(\beta) & 0\\ \sin(\beta) & \cos(\beta) & 0\\ 0 & 0 & 1\end{bmatrix}\mathrm{x}\,\boldsymbol{sp}_{\mathbf{R_1}}$$

$$\Leftrightarrow\qquad \begin{aligned} sp_{R_I}(1) &= \cos(\beta)\cdot sp_{R_1}(1)-\sin(\beta)\cdot sp_{R_1}(2)\\ sp_{R_I}(1) &= \sin(\beta)\cdot sp_{R_1}(1)+\cos(\beta)\cdot sp_{R_1}(2)\\ sp_{R_I}(3) &= sp_{R_1}(3)\end{aligned}$$

7.4. IASI FOOTPRINT

The IFOV of each IASI pixel is delimited by a cone. The cone axis is the viewing direction of the pixel center and its half angle, ε, is a small angle.

The four IASI pixels lie in a cone whose axis is the optical axis, aligned with the axis X_2 of R_2. We now introduce a reference frame R_{4i}, which is similar to the reference frame R_3 used for the conical scanning (defined in 3.6). R_{4i} has its X-axis, aligned with the viewing direction of one IASI pixel M_i and is deduced from R_2 by two rotations:

- rotation of an angle ϕ_i about axis X_2, which gives an intermediate reference frame
- rotation of an angle ζ about the Z-axis of the intermediate reference frame.

The conversion matrix from R_{4i} to R_2, $\mathbf{M_{4i}}$, is similar to the matrix $\mathbf{M_3}$ (defined in 3.6), where the angles δ and ϕ are replaced respectively by ζ and ϕ_I and ζ is a small angle:

$$\mathbf{M_{4i}}=\begin{bmatrix}1 & -\zeta & 0\\ \zeta\cdot\cos(\phi_i) & \cos(\phi_i) & -\sin(\phi_i)\\ \zeta\cdot\sin(\phi_i) & \sin(\phi_i) & \cos(\phi_i)\end{bmatrix}$$

Any viewing direction, ***sp***, lying in the cone that corresponds to the footprint of the IASI pixel M_i, is given in R_{4i} by:

$$\boldsymbol{sp}_{R_{4i}}=\begin{bmatrix}1\\ \varepsilon\cdot\cos(\psi)\\ \varepsilon\cdot\sin(\psi)\end{bmatrix}\qquad 0\le\psi\le 2\pi$$

The viewing vector is converted from R_4 to R_2 with the matrix $\mathbf{M_{4i}}$:

$$\boldsymbol{sp}_{\mathbf{R_2}} = \mathbf{M_{4i}}\ \boldsymbol{sp}_{\mathbf{R_{4i}}} = \begin{bmatrix} 1 & -\zeta & 0 \\ \zeta\cdot\cos(\phi_i) & \cos(\phi_i) & -\sin(\phi_i) \\ \zeta\cdot\sin(\phi_i) & \sin(\phi_i) & \cos(\phi_i) \end{bmatrix} \mathrm{x} \begin{bmatrix} 1 \\ \varepsilon\cdot\cos(\psi) \\ \varepsilon\cdot\sin(\psi) \end{bmatrix}$$

As ε and ζ are both small angles, only the first order terms in ε or ζ are kept:

$$\boldsymbol{sp}_{\mathbf{R_2}} = \begin{bmatrix} 1 \\ \zeta\cdot\cos(\phi_i) + \varepsilon\cdot[\cos(\psi)\cdot\cos(\phi_i) - \sin(\psi)\cdot\sin(\phi_i)] \\ \zeta\cdot\sin(\phi_i) + \varepsilon\cdot[\cos(\psi)\cdot\sin(\phi_i) + \sin(\psi)\cdot\cos(\phi_i)] \end{bmatrix} = \begin{bmatrix} 1 \\ \zeta\cdot\cos(\phi_i) + \varepsilon\cdot\cos(\phi_i+\psi) \\ \zeta\cdot\sin(\phi_i) + \varepsilon\cdot\sin(\phi_i+\psi) \end{bmatrix}$$

Then it is converted from R_2 then to R_I with the matrix $\mathbf{M_2}$:

$$\boldsymbol{sp}_{\mathbf{R_I}} = \mathbf{M_2}\ \boldsymbol{sp}_{\mathbf{R_2}} = \begin{bmatrix} \cos(\alpha_{opt}) & 0 & -\sin(\alpha_{opt}) \\ 0 & 1 & 0 \\ \sin(\alpha_{opt}) & 0 & \cos(\alpha_{opt}) \end{bmatrix} \mathrm{x} \begin{bmatrix} 1 \\ \zeta\cdot\cos(\phi_i) + \varepsilon\cdot\cos(\phi_i+\psi) \\ \zeta\cdot\sin(\phi_i) + \varepsilon\cdot\sin(\phi_i+\psi) \end{bmatrix}$$

$$\boldsymbol{p}_{\mathbf{R_I}} = \begin{bmatrix} \cos(\alpha_{opt}) - \sin(\alpha_{opt})\cdot[\zeta\cdot\sin(\phi_i) + \varepsilon\cdot\sin(\phi_i+\psi)] \\ \zeta\cdot\cos(\phi_i) + \varepsilon\cdot\cos(\phi_i+\psi) \\ \sin(\alpha_{opt}) + \cos(\alpha_{opt})\cdot[\zeta\cdot\sin(\phi_i) + \varepsilon\cdot\sin(\phi_i+\psi)] \end{bmatrix} \qquad 0 \le \psi \le 2\pi$$

8. ORBIT PREDICTION USING 2-LINE ELEMENTS

8.1. RESULTS OF A 6-MONTH COMPARISON

At the Centre de Meteorologie Spatiale (CMS) of Meteo-France, three modes of AAPP navigation have been run under operational conditions, using respectively:

- TBUS bulletins, which is the standard option of AAPP version 1 to 4,
- Two-Line element sets, which is the new option proposed in AAPP-5,
- ARGOS bulletins, which is the CMS operational mode.

Here are presented the results obtained over a 6-month period for NOAA-16 and NOAA-17.

The consistency of each orbital element data set is given by the orbit extrapolation error, which is the distance between the position predicted with the current bulletin and the position of the next bulletin, divided by the time interval between these two bulletins. Statistics of this parameter are presented in Table 8-1 and examples of its temporal variation are shown in the upper plot of Figure 8-1 and Figure 8-2.

Satellite	Method	bias	sigma	r.m.s
noaa16	tbus	-2.92	1.25	3.17
	2line	-0.05	0.76	0.77
	argos	0.61	0.80	1.01
noaa17	tbus	4.70	1.95	5.09
	2line	-0.06	0.66	0.67
	argos	0.78	1.00	1.27

Table 8-1 : extrapolation error in km per day, from 2003/09/22 to 2004/03/15

The results obtained with the Two-Line element sets are obviously better than those obtained with the TBUS bulletins, for both satellites.

The CMS operational suite for AVHRR imagery includes an Automatic Navigation Adjustment (ANA), which is a correlation technique based on coastal landmarks. This allows calculation, for each landmark successfully processed by ANA, of the navigation error as the distance between the AAPP calculated position and the ANA "measured" position. The AAPP position is calculated with a "default" attitude error, which is a parameter introduced by the user (in the satid file) and usually derived from ANA attitude earlier results. The default attitude values used in the 6-month experiment are given in Table 8-2. The default attitude error is calculated separately for the three types of orbital elements, since the so-called "pitch bias" corresponds actually to a pitch error and to an orbit extrapolation error.

Satellite	Method	yaw	roll	pitch
noaa16	tbus	0.2	0.7	-0.6
	2line	0.2	0.7	0.6
	argos	0.2	0.7	0.7
noaa17	tbus	0.8	0.2	4.7
	2line	0.8	0.2	0.9
	argos	0.8	0.2	1.9

Table 8-2 : NOAA-16 and NOOA–17 defaut attitude values (in mrad)

Statistics of the AVHRR navigation error have been calculated over all landmarks of a pass, then over all passes of the 6-month period and the final results are presented in table 3. Examples of the navigation error temporal variation are shown in the plot entitled "distance rms error (km)" of Figure 8-1 and Figure 8-2.

Satellite	Method	bias	sigma	r.m.s
noaa16	tbus	2.39	1.29	2.71
	2line	2.24	0.96	2.44
	argos	2.04	0.82	2.20
noaa17	tbus	3.12	2.27	3.86
	2line	2.09	0.84	2.25
	argos	1.86	0.86	2.04

Table 8-3 : AVHRR navigation error in km, from 2003/09/22 to 2004/03/15, using AAPP default attitude. The statistics are derived from only the passes for which the yaw, roll and pitch have been estimated.

The navigation accuracy with the Two-Line element sets is better than the one with the TBUS bulletins. For NOAA-16, there is only a slight difference, probably because the default pitch error has partly compensated the orbit extrapolation error of the TBUS. For NOAA-17, there is a significant difference.

8.2. CONCLUSION

A new feature, orbit calculation using the Two-Line element sets, has been added to AAPP navigation. The AVHRR imagery is navigated more accurately with these data than with TBUS bulletin. The AAPP user can easily switch from TBUS to Two-Line data and it is recommended to do so.

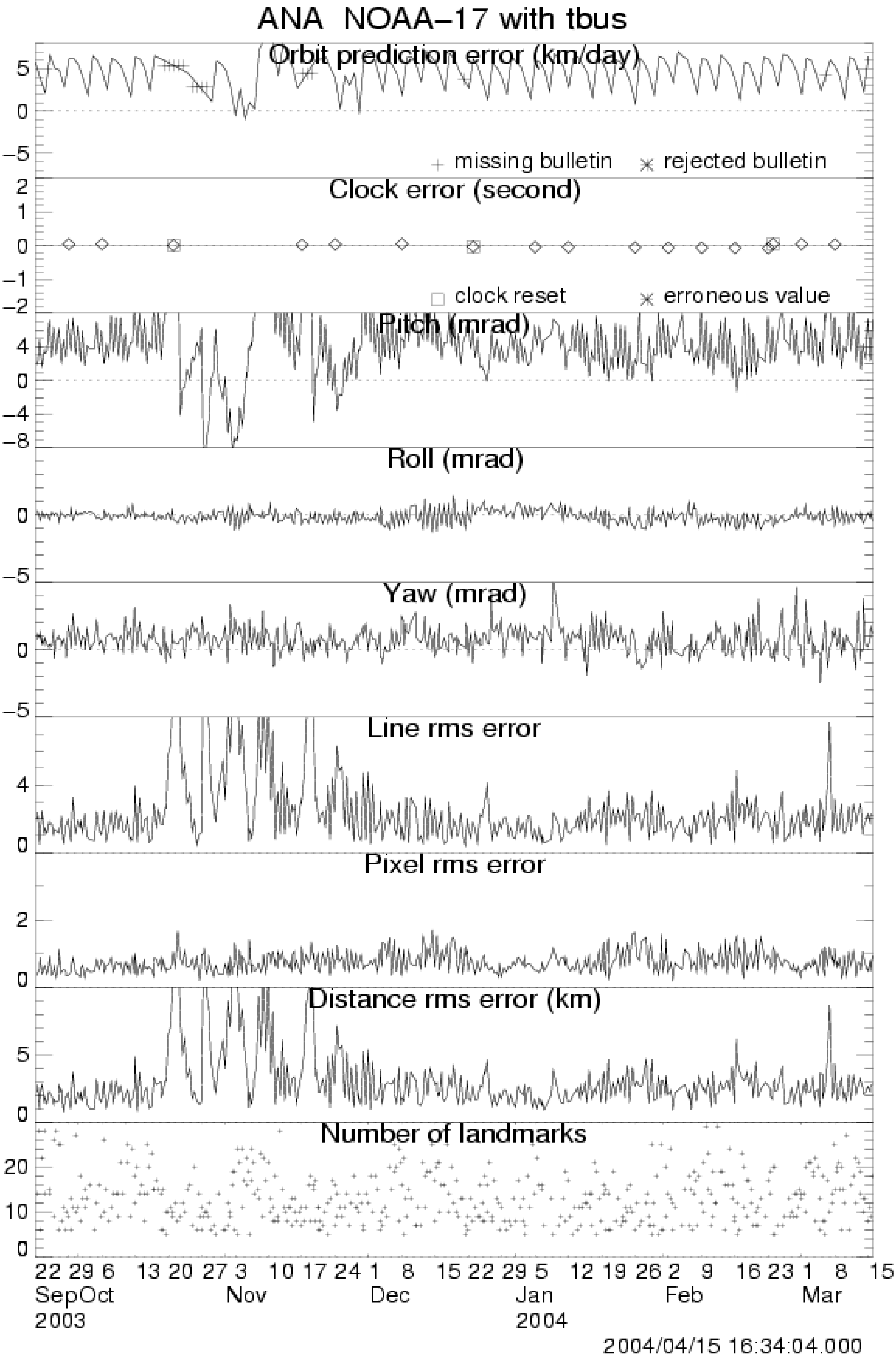

Figure 8-1 : ANA NOAA-17 with TBUS

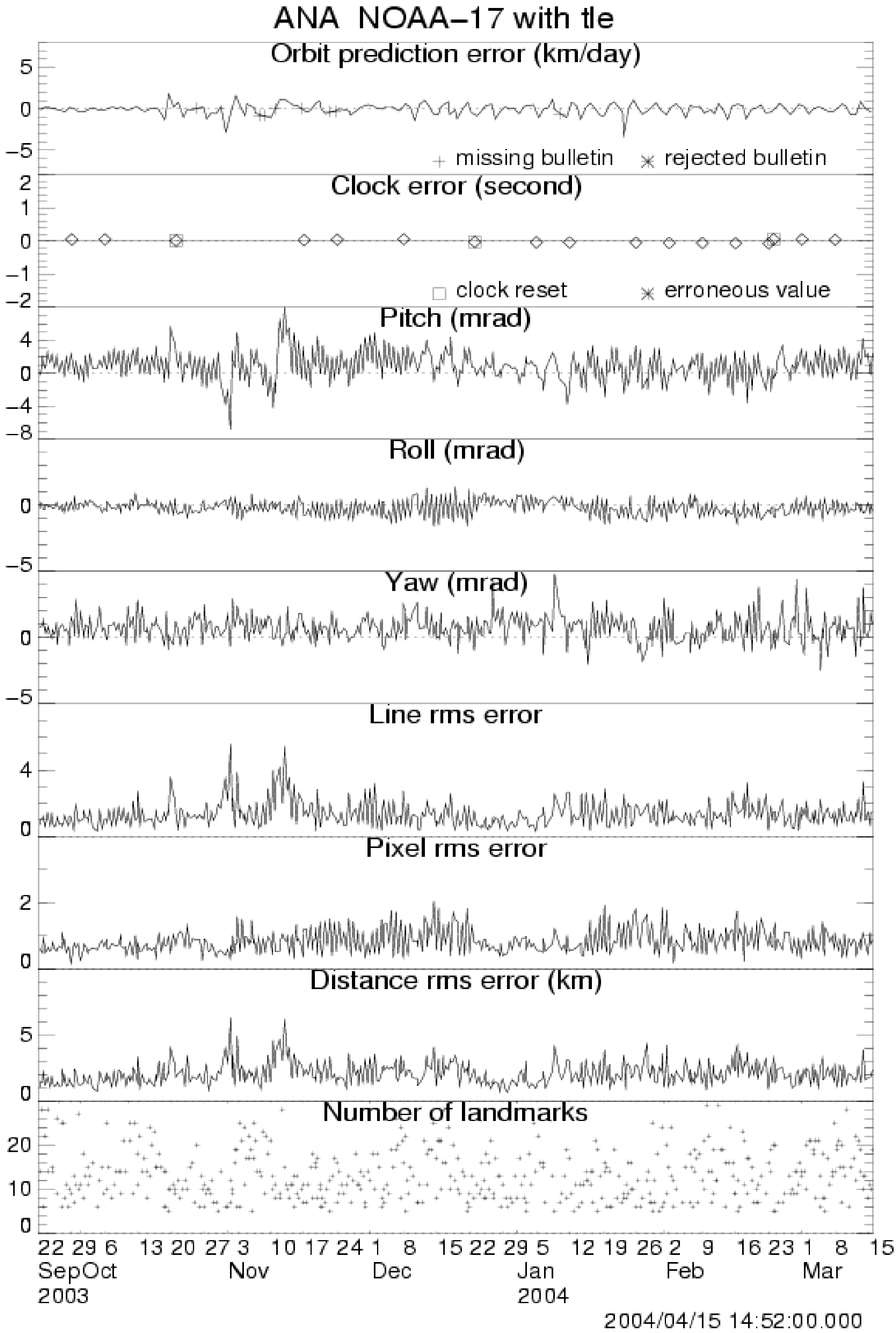

Figure 8-2 : ANA NOAA-17 with TLE